On non-Archimedean valued fields: a survey of algebraic, topological and metric structures, analysis and applications

Khodr Shamseddine

University of Manitoba

In this talk, we first briefly review basic properties of ultrametric spaces, valued fields and ordered fields as well as the connection between these different mathematical objects. As examples, we introduce the so-called general Hahn fields and Levi-Civita fields, and we present a summary of their key properties. Then, we focus our attention on two special Levi-Civita fields: $\mathcal{R}$ and its complex counterpart $\mathcal{C}$. Among all the non-Archimedean fields surveyed in the first part of the talk, $\mathcal{R}$ and $\mathcal{C}$ are unique from a pure Mathematical point of view as well as from a computational point of view. We give a brief summary of our research work on $\mathcal{R}$ and $\mathcal{C}$, with one computational application.